

# The Potential of Interference-Based Proof Systems

Marijn J.H. Heule and Benjamin Kiesl

The University of Texas at Austin and Vienna University of Technology



ARCADE in Gothenburg, Sweden

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## Traditional Proofs vs. Interference-Based Proofs

- In **traditional** proof systems, everything that is **inferred**, is **logically implied** by the premises.

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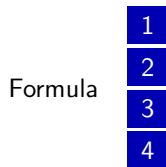
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- ➔ Inference rules reason about the **presence** of facts.
  - If certain premises are present, infer the conclusion.
- **Different approach**: Allow **not only implied conclusions**.
  - **Require only** that the addition of facts preserves **satisfiability**.
  - Reason also about the **absence** of facts.
- ➔ This leads to **interference-based proof systems**.

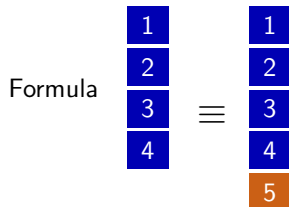
# Interference-Based Proof Systems

- **Interference-based proof systems** generalize traditional proof systems.
- An **interference-based proof** is a sequence of clauses.
  - **Idea**: Clauses are added to the formula or deleted from it step-by-step.
  - Added clauses need not be implied, but their addition must preserve **satisfiability**:
    - ➔ If the formula is satisfiable, then the formula obtained by adding a clause is also satisfiable.
    - ➔ If the (unsatisfiable) **empty clause**,  $\perp$ , can be added, then the original formula must be **unsatisfiable**.
      - ▶ The **empty clause is unsatisfiable** because it has no literal that could be true.

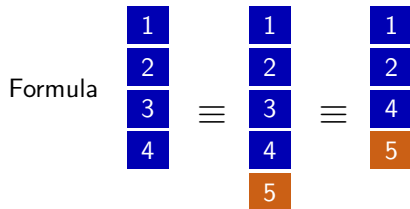
# Interference-Based Proofs



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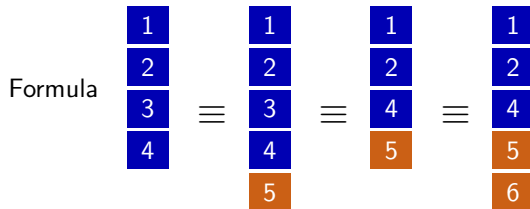


# Interference-Based Proofs





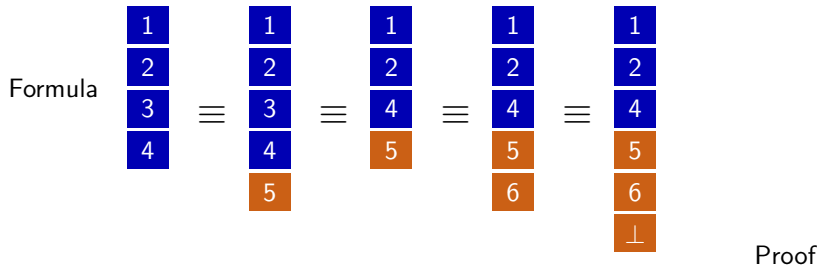
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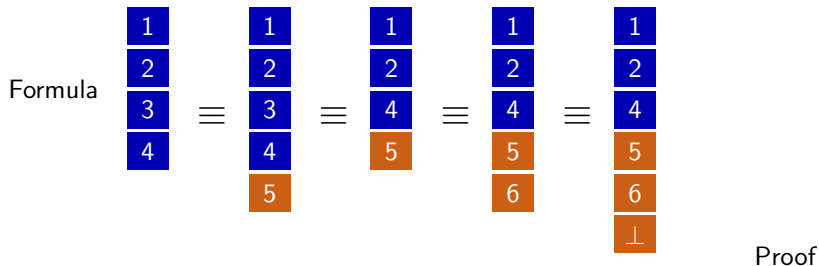
Proof



# Interference-Based Proofs



# Interference-Based Proofs



- Checking whether modifications preserve satisfiability should be **efficient**.
- Clauses that can be added or removed are called **redundant**.
- ➔ **Idea:** Allow only modifications that fulfill an **efficiently checkable redundancy criterion**.
- ➔ **Idea:** Showing satisfiability equivalence with the empty formula allows **proving satisfiability**.

# Practical Usefulness of Interference-Based Proof Systems

- The interference-based proof system **DRAT** is the **de-facto standard in SAT solving**.
  - DRAT is based on the addition of RAT clauses and on deletion.
- **QRAT**, the extension of DRAT for the satisfiability problem of **quantified Boolean formulas (QSAT)**, is very powerful.
  - It is the only QBF proof system that can succinctly express **virtually all preprocessing techniques**.
- At CADE we present new interference-based proof systems for SAT that allow for **short proofs without new variables**.

# Question 1

- Almost all proof systems reason only about the **presence** of premises.
- *What prevents us, in particular theoreticians, from reasoning about their **absence**?*
- The best known interference-based system, **extended resolution**, allows exponentially smaller proofs compared to resolution.
- Other interference rules such as **blocked-literal addition** in QSAT also facilitate **short proofs** for hard problems.

## Question 2

- Deletion of clauses can be a powerful technique.
- In SAT, clause deletion provides **efficiency**.
- In QSAT, it provides a way to **prove satisfiability**.
- *What can clause deletion offer in **first-order logic**?*

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